Two-Field Q-ball Solutions of Supersymmetric Hybrid Inflation

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Abstract

We demonstrate the existence of two-field Q-ball solutions of the scalar field equations of supersymmetric D- and F-term hybrid inflation. The solutions consist of a complex inflaton field together with a real symmetry breaking field. Such inflatonic Q-balls may play a fundamental role in reheating and the post-inflation era of supersymmetric hybrid inflation models.

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1 Introduction

Hybrid inflation models [1, 2] are a favoured class of inflation model, being able to account for both a flat inflaton potential during inflation and for a massive inflaton and reheating after inflation, without requiring very small couplings. In supersymmetry (SUSY) there are two classes of hybrid inflation model, D-term inflation [3] and F-term inflation [4], depending on whether the energy density driving inflation originates in a D-term or F-term contribution to the scalar potential.

An important period in early Universe cosmology is the era immediately following the end of inflation, the post-inflation era. Important physical processes such as baryogenesis are likely to occur during the post-inflation era, whilst reheating of the Universe will occur at the end of this era. In hybrid inflation models it is known that quantum fluctuations of the inflaton sector fields will rapidly grow and become non-linear at the end of inflation [5, 6, 7, 8]. The question of the subsequent evolution of the non-linear field configurations in realistic SUSY inflation models remains to be fully explored, but one possibility is that non-topological soliton configurations will form [6, 7, 9]. The most stable such configuration will tend to be the dominant one, since this will come to dominate the energy density of the Universe and so determine the physics of the post-inflation era.

In SUSY hybrid inflation models the scalar fields are generally complex, and therefore can carry conserved global $U(1)$ charges. Depending on the form of the scalar potential, it is then possible that a Q-ball made of inflaton sector fields exists [10]. If such Q-balls formed at the end of inflation, the Universe following inflation would be highly inhomogeneous, with all the energy density concentrated in the form of inflatonic Q-balls. Post-inflation physics would then take place against this cosmological background [6, 8, 11], whilst reheating would occur via the eventual decay of the Q-balls[1].

The purpose of this paper is to demonstrate the existence of inflatonic Q-balls in SUSY hybrid inflation models. We will present numerical examples of two-field Q-ball

[1]This possibility has previously been realised in the context of a single field chaotic inflation model [12].

solutions, composed of a complex inflaton field carrying a global $U(1)$ charge and a real symmetry breaking field.

The paper is organised as follows. In Section 2 we discuss SUSY hybrid inflation models and the Q-ball equations. In Section 3 we demonstrate the existence of two field Q-ball solutions of these equations for the case of D- and F-term inflation. In Section 4 we present our conclusions.

2 Q-ball Equations of SUSY Hybrid Inflation

SUSY hybrid inflation models are either F-term or D-term models. The simplest F-term inflation model has a superpotential of the form [2, 4]

$$W = \frac{\eta}{2} S(\Phi^2 - \mu^2) \ , \tag{1}$$

where S is the inflaton and Φ is a field which gains an expectation value which terminates inflation. μ^2 and η are real and positive. The scalar potential is then

$$V = \eta^2 |S|^2 |\Phi|^2 + \frac{\eta^2}{4} |\Phi^2 - \mu^2|^2 \ . \tag{2}$$

The scalar potential has an R-symmetry under which only S transforms, which manifests itself as a global $U(1)$ symmetry in the scalar potential, $S \rightarrow e^{i\alpha} S$, with respect to which we can define a conserved global charge.

D-term inflation models have a superpotential of the form [3]

$$W = \lambda S \Phi_+ \Phi_- \ . \tag{3}$$

The scalar potential is given by,

$$V = \lambda^2 |S|^2 \left(|\Phi_+|^2 + |\Phi_-|^2 \right) + \lambda^2 |\Phi_+|^2 |\Phi_-|^2 + \frac{g^2}{2} \left(|\Phi_+|^2 - |\Phi_-|^2 + \xi \right)^2 \ , \tag{4}$$

where S is the inflaton, $\Phi_\pm$ are fields with charges ± 1 with respect to a Fayet-Illiopoulos $U(1)_{FI}$ gauge symmetry, $\xi > 0$ is the Fayet-Illiopoulos term and g is the $U(1)_{FI}$ gauge coupling.

The D-term inflation scalar potential is a function of $|S|$, $|\Phi_+|$ and $|\Phi_-|$ and therefore has three global $U(1)$ symmetries, $S \to e^{i\alpha}S$ ($U(1)_S$), $\Phi_+ \to e^{i\beta_+}\Phi_+$ ($U(1)_+$) and $\Phi_- \to e^{i\beta_-}\Phi_-$ ($U(1)_-$). We can define conserved charges with respect to these global $U(1)$ symmetries, Q_S, Q_+ and Q_-.

Since SUSY hybrid inflation models have conserved global charges, it is possible that there exist Q-balls [10]. We first review the case of a single complex field Φ with $U(1)$ symmetry $\Phi \to e^{i\alpha}\Phi$. The Q-ball configuration is derived by minimizing the energy whilst fixing the charge via a Lagrange multiplier i.e. by minimizing the functional [13]

$$E_\omega = E + \omega\left(Q - \int d^3x \rho_Q\right) \ , \tag{5}$$

with respect to the scalar fields and ω, where E is the total energy of the field configuration

$$E = \int d^3x \ |\dot{\Phi}|^2 + |\underline{\nabla}\Phi|^2 + V(|\Phi|) \tag{6}$$

and ρ_Q is the charge density

$$\rho_Q = i(\dot{\Phi}^\dagger\Phi - \Phi^\dagger\dot{\Phi}) \ . \tag{7}$$

E_ω may be equivalently written as

$$E_\omega(\dot{\Phi}, \Phi, \omega) = \int d^3x \left(|\dot{\Phi} - i\omega\Phi|^2 + |\underline{\nabla}\Phi|^2 + V(\Phi) - \omega^2|\Phi|^2\right) + \omega Q \ . \tag{8}$$

This should be minimized with respect to $\dot{\Phi}$, Φ and ω. To minimize with respect to $\dot{\Phi}$ we require $\Phi(\mathbf{x}, t) = \Phi(\mathbf{x})e^{i\omega t}$. Substituting this into Eq. (8) gives

$$E_\omega(\Phi(\mathbf{x}), \omega) = \int d^3x \left(|\underline{\nabla}\Phi(\mathbf{x})|^2 + V(\Phi(\mathbf{x})) - \omega^2|\Phi(\mathbf{x})|^2\right) + \omega Q \ . \tag{9}$$

Extremizing this with respect to $\Phi(\mathbf{x})$ implies that

$$\underline{\nabla}^2\Phi(\mathbf{x}) = \frac{\partial V_\omega\left(\Phi(\mathbf{x})\right)}{\partial\Phi^\dagger} \tag{10}$$

where $V_\omega = V - \omega^2|\Phi|^2$. At this point $\Phi(\mathbf{x})$ could still have a space-dependent complex phase, $\theta(\mathbf{x})$. If $V(\Phi) = V(|\Phi|)$, as it must when Φ transforms under a $U(1)$ symmetry, then Eq. (9) is generally minimized by the choice $\theta = \mathit{constant}$, which may be

chosen such that $\Phi(\mathbf{x})$ is real. A minimum energy configuration should be spherically symmetric. Then, with $\Phi(\mathbf{x}) = \phi(r)/\sqrt{2}$ ($\phi(r)$ real), Eq. (10) becomes

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r}\frac{\partial \phi}{\partial r} = \frac{\partial V}{\partial \phi} - \omega^2 \phi \ . \tag{11}$$

We refer to this as the Q-ball equation. The solutions of Eq. (11) should satisfy the boundary conditions that the field tends to the vacuum as $r \to \infty$ and that $\partial\phi/\partial r \to 0$ as $r \to 0$.

The above analysis generalizes to the multiple complex scalar field case of SUSY hybrid inflation. For each value of ω there can be many solutions of the Q-ball equations satisfying the boundary conditions, each with a different energy and charge. In particular, there will be a range of solutions of different energy and ω for a given global charge. Each of these solutions is a Q-ball in the sense that it is a solution of the scalar field equations corresponding to a non-topological soliton which has a time-independent amplitude as a function of r. However, these Q-balls will be metastable with respect to the lowest energy Q-ball solution. The stable Q-ball solution is the lowest energy field configuration for a given global charge, obtained by minimizing the energy functional with respect to ω for a fixed charge.

We will refer to solutions of Eq. (11) which are not minimum energy solutions for a given charge as 'metastable Q-balls'. The existence of one metastable Q-ball solution for a given charge is sufficient to prove the existence of the stable Q-ball; it is either the minimum energy solution itself or there exists a lower energy solution of Eq. (11) carrying the same global charge.

In the following we will focus on the case of D-term inflation. This is because, as we will show, the Q-ball equations for the case of F-term inflation are equivalent to those of D-term inflation with $\lambda = \sqrt{2}g$. Therefore the F-term inflation Q-balls are a subset of those of D-term inflation.

We now consider three Lagrange multipliers ω, γ_+ and γ_-, corresponding to the conserved charges Q_S, Q_+ and Q_- respectively. The functional is now

$$E_\omega = E + \omega \left(Q_S - \int d^3x \rho_{Q_S} \right) + \gamma_+ \left(Q_+ - \int d^3x \rho_{Q_+} \right) + \gamma_- \left(Q_- - \int d^3x \rho_{Q_-} \right) \ , \tag{12}$$

where

$$E = \int d^3x \; |\dot{S}|^2 + |\underline{\nabla} S|^2 + |\dot{\Phi}_+|^2 + |\underline{\nabla}\Phi_+|^2 + |\dot{\Phi}_-|^2 + |\underline{\nabla}\Phi_-|^2 + V(|S|,|\Phi_+|,|\Phi_-|) \;, \quad (13)$$

$$\rho_{Q_S} = i(\dot{S}^{\dagger} S - S^{\dagger}\dot{S}) \quad (14)$$

and

$$\rho_{Q_\pm} = i(\dot{\Phi}_\pm^{\dagger}\Phi_\pm - \Phi_\pm^{\dagger}\dot{\Phi}_\pm) \;. \quad (15)$$

As before, minimizing the time derivative terms implies that $S(\mathbf{x},t) = S(\mathbf{x})e^{i\omega t}$ and $\Phi_\pm(\mathbf{x},t) = \Phi_\pm(\mathbf{x})e^{i\gamma_\pm t}$. For D-term inflation, $V = V(|S|,|\Phi_+|,|\Phi_-|)$. Therefore the minimum energy configuration will correspond to real $S(\mathbf{x})$ and $\Phi_\pm(\mathbf{x})$. Assuming a spherically symmetric minimum energy configuration then implies that $S = s(r)e^{i\omega t}/\sqrt{2}$ and $\Phi_\pm = \phi_\pm(r)e^{i\gamma_\pm t}/\sqrt{2}$. The vacuum of D-term inflation corresponds to $|\Phi_-| = \xi^{1/2}$ and $S = \Phi_+ = 0$. Therefore we must have $\gamma_- = 0$ in order to reach the Φ_- vacuum expectation value as $r \rightarrow \infty$. Thus the D-term inflation Q-ball must have $Q_- = 0$. Since $\Phi_+ \rightarrow 0$ as $r \rightarrow \infty$, the Q-ball could, in principle, carry a $U(1)_+$ charge. However, it is unlikely that a Q-ball solution with a $U(1)_+$ charge exists. This is because the effective mass of the Φ_+ scalar increases as $|\Phi_-|$ decreases from ξ and S increases from zero as $r \rightarrow 0$. Thus a $U(1)_+$ charge is likely to be energetically disfavoured. Therefore we will focus on the case $Q_+ = 0$, for which $\gamma_+ = 0$.

The corresponding Q-ball equations are then

$$\frac{\partial^2 s}{\partial r^2} + \frac{2}{r}\frac{\partial s}{\partial r} = \frac{\lambda^2}{2}\left(\phi_+^2 + \phi_-^2\right) s - \omega^2 s \;, \quad (16)$$

$$\frac{\partial^2 \phi_+}{\partial r^2} + \frac{2}{r}\frac{\partial \phi_+}{\partial r} = \frac{\lambda^2}{2}\left(s^2 + \phi_-^2\right)\phi_+ + g^2\left(\xi - \frac{\phi_-^2}{2}\right)\phi_+ + \frac{g^2}{2}\phi_+^3 \quad (17)$$

and

$$\frac{\partial^2 \phi_-}{\partial r^2} + \frac{2}{r}\frac{\partial \phi_-}{\partial r} = \frac{\lambda^2}{2}\left(s^2 + \phi_+^2\right)\phi_- - g^2\left(\xi + \frac{\phi_+^2}{2}\right)\phi_- + \frac{g^2}{2}\phi_-^3 \;. \quad (18)$$

For $\phi_-^2 \leq 2\xi$, which will be true for any solution tending to the vacuum as $r \rightarrow \infty$, the only solution of Eq. (17) which satisfies the boundary conditions $\phi_+ \rightarrow 0$ as $r \rightarrow \infty$ and $\partial\phi_+/\partial r \rightarrow 0$ as $r \rightarrow 0$ is $\phi_+(r) = 0 \;\forall r$. This follows since for a minimum energy solution we expect $\partial\phi_+/\partial r \leq 0$, such that $\phi_+(r)$ is monotonically decreasing to zero

as r increases. Since the right hand side of Eq. (17) is positive $\forall r$, it then follows that $\partial^2\phi_+/\partial r^2 > 0 \; \forall r$. However, for a monotonically decreasing $\phi_+(r)$ we require that $\partial^2\phi_+/\partial r^2 < 0$ at $r = 0$. Therefore there is no non-trivial solution. Thus for a Q-ball with $Q_S \neq 0$ and $Q_+ = 0$, the Q-ball equations become

$$\frac{\partial^2 s}{\partial r^2} + \frac{2}{r}\frac{\partial s}{\partial r} = \frac{\lambda^2}{2}\phi_-^2 s - \omega^2 s \tag{19}$$

and

$$\frac{\partial^2 \phi_-}{\partial r^2} + \frac{2}{r}\frac{\partial \phi_-}{\partial r} = \left(\frac{\lambda^2}{2}s^2 - g^2\xi\right)\phi_- + \frac{g^2}{2}\phi_-^3 \; . \tag{20}$$

Therefore the Q-ball solution of D-term inflation with $Q_+ = 0$ consists of a complex S field and a real Φ_- field, with a Q_S charge but no Q_- charge. The energy and charge of the resulting Q-ball soultion are given by

$$E = \int 4\pi r^2 dr \left[\frac{1}{2}\left(\frac{\partial s}{\partial r}\right)^2 + \frac{1}{2}\left(\frac{\partial \phi_-}{\partial r}\right)^2 + \frac{\omega^2 s^2}{2} + V(s,\phi_-)\right] \tag{21}$$

and

$$Q_S = \omega \int 4\pi r^2 dr \; s^2 \; . \tag{22}$$

In the case of F-term inflation, only the S field can carry a global charge. Therefore, upon performing the minimization of the energy functional, from minimizing with respect to $\dot{S}$ and $\dot{\Phi}$ we obtain $S(\mathbf{x},t) = S(\mathbf{x})e^{i\omega t}$ and $\Phi(\mathbf{x},t) = \Phi(\mathbf{x})$. The potential, Eq. (2), is explicitly dependent upon the phase of Φ. However, for μ^2 real and positive the energy functional will still be minimized by having both $S(\mathbf{x})$ and $\Phi(\mathbf{x})$ real and positive. Therefore with $S(\mathbf{x}) = s(r)/\sqrt{2}$ and $\Phi(\mathbf{x}) = \phi(r)/\sqrt{2}$, the Q-ball equations become

$$\frac{\partial^2 s}{\partial r^2} + \frac{2}{r}\frac{\partial s}{\partial r} = \frac{\eta^2}{2}\phi^2 s - \omega^2 s \tag{23}$$

and

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r}\frac{\partial \phi}{\partial r} = \left(\frac{\eta^2}{2}s^2 - \frac{\eta^2}{2}\xi\right)\phi + \frac{\eta^2}{4}\phi^3 \; . \tag{24}$$

These equations are the same as the two-field D-term inflation equations, Eq. (19) and Eq. (20), when $\phi_- \leftrightarrow \phi$, $\lambda \leftrightarrow \eta$ and $g \leftrightarrow \eta/\sqrt{2}$ i.e. when $\lambda = \sqrt{2}g$.

3 Numerical Two-Field Q-Ball Solutions

In this section we will present a number of numerical solutions of Eq. (19) and Eq. (20) corresponding to Q-balls. We consider $g = 1$ throughout.

For a given λ there will be a range of solutions corresponding to different values of Q_S. In Figure 1 we show a Q-ball solution for $s(r)$ and $\phi_{-}(r)$ for the case $\lambda = 0.5$. (We use units such that $\xi = 1$.) In Figure 2 we show a solution for $\lambda = 1$. In Figure 3 we show a solution for the special case $\lambda = \sqrt{2}g$, corresponding to the case of F-term inflation. In Table 1 we summarise the properties of these example Q-ball solutions. (We define the radius as the value of r within which 90% of the total Q-ball energy is contained.) E/Q_S is less than the S mass in vacuum for all of these solutions ($m_S = \lambda\xi^{1/2} \equiv \lambda$ in our units), so in the absence of additional couplings to the MSSM fields the Q-balls will be absolutely stable as a result of Q_S conservation. (However, once the (unknown) couplings of the inflaton sector fields to the MSSM fields are included, the inflatonic Q-balls will decay to MSSM fields via conventional inflaton decay.) An interesting feature of these solutions is that the value of $s(r = 0)$ at the centre of the Q-balls is *larger* than the value at which the symmetry breaking phase transition ending inflation occurs, $s(r = 0) > s_c = \sqrt{2}g\xi^{1/2}/\lambda$ ($\equiv 1.41g/\lambda$ for $\xi = 1$). However, in the Q-ball solution, where the field configuration is dependent upon the gradient energy as well as the potential energy, the symmetry breaking field, ϕ_{-}, remains non-zero throughout the Q-ball.

λ	ω	s_o	ϕ_o	E	Q_S	E/Q_S	r
0.5	0.375	0.20	0.00371	2894	6346	0.456	8.31
1.0	0.800	3.00	0.14357	378	396	0.956	4.33
$\sqrt{2}$	1.125	2.44	0.19680	180	132	1.368	3.11

Table 1: Properties of example Q-ball solutions for $g = 1$.

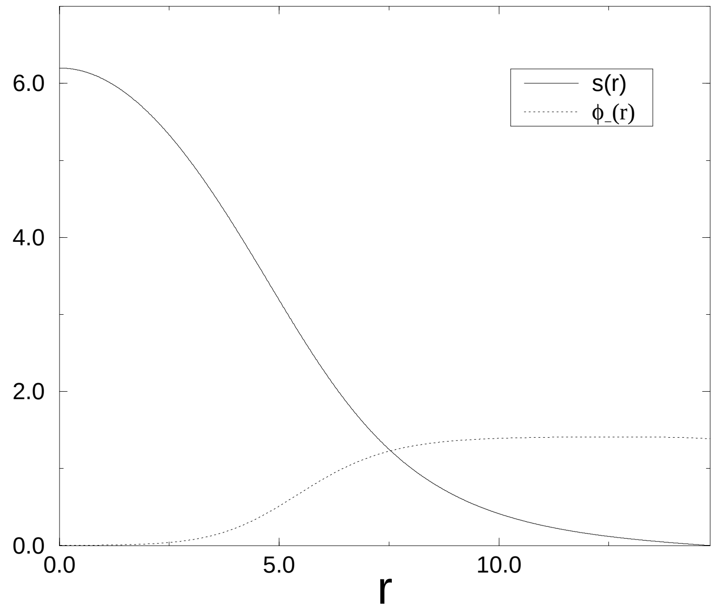

Figure 1: Q-ball profile for $\lambda = 0.5$ and $g = 1$.

In Table 2 we give the values of ω, $s(r=0)$, $\phi_-(r=0)$, r and E/Q_S for metastable Q-balls with $\lambda = 1$, $g = 1$ and fixed charge $Q_S \approx 395$. Since the value of Q_S is generally much larger than 1, the Q-balls may be studied classically. Metastable Q-ball solutions exist for a finite range of ω. The lowest value of E/Q_S corresponds to the true Q-ball solution. As ω decreases, the value of E/Q_S decreases, implying that the binding energy of the S charges in the Q-ball is increasing. An interesting feature is that the value of s at the centre of the Q-ball increases whilst r decreases as E/Q_S increases, indicating that the metastable Q-balls have a larger gradient energy than the true Q-balls for a given charge.

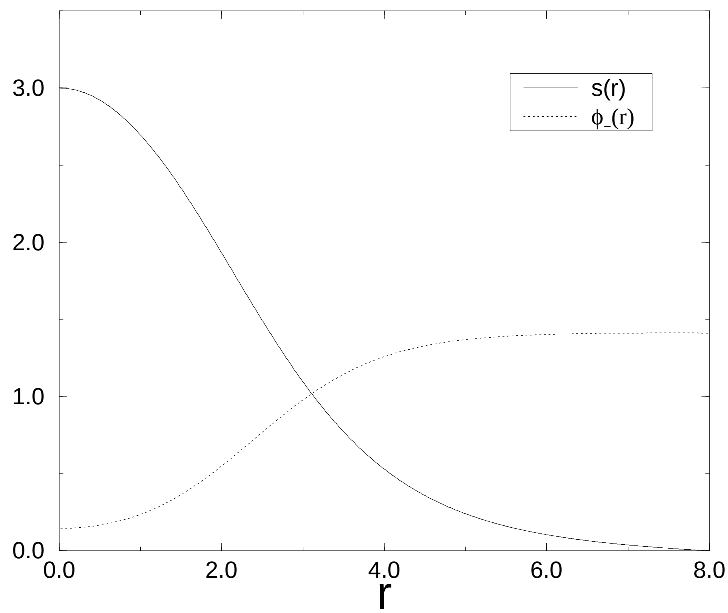

Figure 2: Q-ball profile for $\lambda = 1$ and $g = 1$.

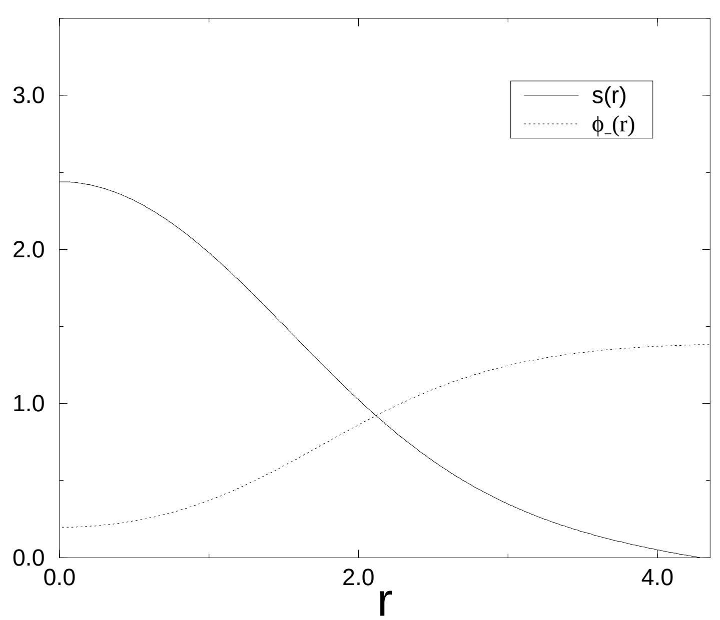

Figure 3: Q-ball profile for $\lambda = \sqrt{2}$ and $g = 1$ (F-term inflation).

ω	s_o	ϕ_o	r	E/Q_S
0.800	3.00	0.1436	4.33	0.956
0.810	3.18	0.1160	4.07	0.978
0.815	3.20	0.1153	4.03	0.981
0.820	3.26	0.1082	3.94	0.982
0.825	3.30	0.1044	3.92	0.991
0.849	3.48	0.0903	3.65	0.999

Table 2: Table of properties of metastable Q-balls for $\lambda = 1$ and $g = 1$.

4 Conclusions

We have provided examples of inflatonic Q-ball solutions of the SUSY D-term hybrid inflation scalar field equations for typical values of the dimensionless couplings λ and g, including the special case $\lambda = \sqrt{2}g$, corresponding to F-term inflation. Since $E/Q_S < m_S$ for these solutions, Q_S conservation implies that the Q-balls are stable up to the decay of the inflaton sector particles they are made of to particles in the minimal SUSY Standard Model sector. Inflatonic Q-balls may form at the end of SUSY hybrid inflation via the formation of neutral condensate lumps and their subsequent decay into Q-ball, anti-Q-ball pairs. In this case there would be a highly inhomogeneous post-inflation era, with the energy density of the Universe concentrated inside the Q-balls and reheating via Q-ball decay. We hope to discuss in detail the classically stable Q-ball solutions and the process of Q-ball formation following SUSY hybrid inflation in future work.

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